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THE PERIODICALLY KICKED TWO-LEVEL ATOM

Amador Muriel
National Institute of Physics
University of the Philippines, Diliman, Quezon City, Philippines

ABSTRACT

The Liouville equation for the two-level atom is solved exactly for the case of a periodic delta-dunction driving field.

Introduction

One of the most studied quantum systems is the two-level atom driven by an external field(1-11). Some results show the following phenomena: periodic collapse and growth of the population difference between the two levels(2), and the occurrence of chaos in a two-level system coupled to an external field described by Maxwell's equations(11).

Because of the advent of picosecond lasers(12) and the interest in intense driving fields(13), it would useful to describe the behavior of a two-level atom in a periodic delta-function pulse of the form $a(t) = E\delta(t-m\tau)$ where E is the amplitude of an external field of period τ pulsed as delta function. m is an integer from 0 to infinity. This analysis continues a pioneering study by Casati *et al.*(14), which was continued by Milonni *et al.*(15), who introduced quantum models driven by external delta-function kicks.

Let k_{ij} represent the elements of the density matrix for the system. In conventional notation, the Liouville equation for this system is written as

$$i\frac{\partial}{\partial t}\begin{matrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{matrix} = \begin{matrix} \omega_1 & -pa/\hbar \\ -pa/\hbar & \omega_2 \end{matrix} , \begin{matrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{matrix} , \quad (1)$$

where h is the Planck constant, p is the dipole moment, and the r.h.s. represents the usual commutator notation. a is the external driving field. $\hbar\omega_1$ and $\hbar\omega_2$ represent the energy of the two levels. Eq. (1) is a reformulation of the periodically kicked quantum system proposed by Milonni *et al.*(15), which we now solve exactly.

Defining the variables $\omega = \omega_1 - \omega_2$, $x = k_{11} - k_{22}$, $y = k_{12} - k_{21}$, $z = k_{12} + k_{21}$, we can transform eq. (1) to the Bloch form

$$i \frac{\partial x}{\partial t} = \alpha a(t) y, \quad i \frac{\partial y}{\partial t} = \omega z + \alpha a(t) x, \quad i \frac{\partial z}{\partial t} = \omega y, \tag{2}$$

where we have put $\alpha = 2pE/\hbar$. Let $y = iu$. Eq. (2) becomes

$$\frac{\partial x}{\partial t} = \alpha a(t) u, \quad \frac{\partial u}{\partial t} = -\omega z - \alpha a(t) x, \quad \frac{\partial z}{\partial t} = \omega u, \tag{3}$$

which incidentally results in $x^2 + u^2 + z^2 = \text{const.}$[11].

Let us put $a(t) = E\ \Sigma_{m=0}^{M}\ \delta(t - m\tau)$ where int (t/τ) is an integer M defined by the range $M < t/\tau \leqslant M + 1$.

Taking the Laplace transforms of eq. (3), and evaluating all integrals with a delta-function, we arrive at algebraic relations for the transforms. Inverting the transforms, we get the following results:

$$x(t) = x(0) + u(0) \sum_{m=0}^{M} \cos(m\tau\omega) + 2\alpha^2\omega \sum_{m=0}^{M} \sum_{k=0}^{M} x(k\tau) \sin[\omega(t - k\tau)], \tag{4}$$

$$u(t) = u(0)\cos\omega t - z(0)\sin\omega t + z(0)(\cos\omega t - 1) - 2\alpha\omega \sum_{m=0}^{M} x(m\tau) \sin[\omega(t - m\tau)], \tag{5}$$

$$z(t) = z(0) + u(0)\sin\omega t + z(0)(\cos\omega t - 1) - 2\alpha\omega \sum_{m=0}^{M} x(m\tau) (\cos[\omega(t - m\tau)] - 1), \tag{6}$$

which complete the solution of the Liouville equation (1). The solution may be verified by straightfoward differentiation using the relation

$$\frac{\partial}{\partial t} \sum_{m=0}^{M} f(m) = f(t) \sum_{m=0}^{M} \delta(t - m\tau),$$

where M has been defined above.

For an alternating field, we can put $(-1)^m$ in front of every summation over the variable m.

When $u(0) = z(0) = 0$, eq. (4) becomes

$$x(t) = x(0) + 2\alpha^2\omega \sum_{m=0}^{M} \sum_{k=0}^{M} x(k\tau) \sin[\omega(t - k\tau)[. \tag{7}$$

For time steps in units of τ, such that $t = p\tau$, we get

$$x(p) = x(0) + 2\alpha^2\omega \sum_{q=0}^{p} qx[(p - q)\tau] \sin(q\omega\tau). \tag{8}$$

The non-markovian nature of the solution is explicitly displayed in eq. (8), and provides an analytic, exact confirmation of the numerical results of Milonni *et al.*[15] which showed that the kicked two-level atom retains a memory of the initial state. We also see no evidence for chaotic behavior in this example of a periodically kicked two-level system.

References

1. E.T. Jaynes and F.W. Cummings, Proc. IEEE 51 (1963) 89.
2. J.H. Eberly, N.B. Narozhny and J.J. Sanchez-Mondragon, Phys. Rev. Lett. 44 (1980) 1323.
3. N.B. Narozhny, J.J. Sanchez-Mondragon and J.H. Eberly, Phys. Rev. A 23 (1981) 236.
4. P.L. Knight and P.M. Radmore, Phys, Lett. A 90 (1982 342.
5. S. Singh, Phys. Rev. A 25 (1982) 3206.
6. B. Buck and C.V. Sukumar, Phys. Lett. A 81 (1981) 132.
7. B. Buck and C.V. Sukumar, Phys. Lett. A 83 (1983) 211.
8. B. Buck and C.V. Sukumar, J. Phys A. 17 (1984) 877.
9. B. Buck and C.V. Sukumar, J. Phys. A 17 (1984) 885.
10. L. Allen and J.H. Eberly, Optical resonance and two-level atom (Wiley, New York, 1975).
11. P. Milonni, J.R. Ackerhalt an H.W. Galbraith, Phys. Rev. Lett. 50 (1983) 966.
12. J. Rothenburg, D. Grischkowsky and A.C. Balant, Phys. Rev. Lett. 53 (1984) 552.
13. P. Lambropoulos and S.J. Smith, eds., Multiphoton processes (Springer, Berlin, 1984).
14. G Casati, B.V. Chirikov, F.M. Izraelev and J. Ford, Lecture notes in physics. Vol. 93. Stochastic behavior in classifical and quantum hamiltonian systems (Sprainger, Berlin, 1979).
15. P.W. Milonni, J.R. Ackerhalt and M.E. Goggin, Phys. Rev. A 35 (1987) 1714.