

A Mean Field RVB Theory For Copper Oxide-Based High T_c Superconductors in Terms of Auxiliary Bosons

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ABSTRACT

A mean-field theory for Anderson's resonating valence bond (RVB) model of high temperature superconductivity in terms of auxiliary boson fields is discussed in this paper. Contact with the conventional Bardeen-Cooper-Schrieffer (BCS) theory in the low temperature limit is achieved through an appropriate choice of order parameters. By devising an effective Hamiltonian in momentum space, holon condensation is shown to occur. This phenomenon triggers superconductivity.

PRELIMINARIES

Sometime in 1986 (1) a synthesis of a complicated ceramic compound of four elements (La-Ba-Cu-O) and the subsequent detection of the appearance of superconductivity at a temperature of 35K generated feverish interests in the scientific world. The raising of the critical temperature to around 95K by Chu and collaborators (2) by working on another ceramic compound (Y-Ba-Cu-O) intensified these interests. Hundreds of scientists all over the world are racing to reach higher critical temperatures by investigating other ceramic compounds; the highest obtained so far is 120K with thallium as the main element. But reports on other superconductors made of organic compounds seem to indicate that the critical temperature could be raised some more. There is no doubt that a new vista in the field of physics has been opened for exploration.

Superconductivity (4) at low critical temperature is well understood. The BCS theory consistently explains the properties of low T_c superconductors through the attractive electron-phonon interaction that provides the mechanism for the formation of bound cooper pairs. But it seems there is an upper bound critical temperature for the BCS theory to be workable.

There are remarkable features of high T_c superconductors that require explanations other than the conventional electron-phonon mechanism. For instance, some experiments seem to indicate that electrons are paired with an energy gap but that no isotopic effect was detected. This certainly could not be explained by the electron-phonon interaction. The mechanism involved might be due to the electron-plasmon attractive force as speculated by some investigators. In the subsequent sections we shall see some more of these as we unravel a mechanism which we believe might be a viable explanation for the behavior and characteristics of high T_c superconductors.

A direct offshoot of investigations of high T_c superconductors in the copper oxide-based ceramics is that it gives us insight into the unexpected magnetic and transport properties of pure as well as doped Mott insulators. Mott insulators, which insulate solely through the coulomb interactions, have never been quite understood for sometime. The resurgence of interest in the Mott localization for strongly correlated systems appears to confirm that this phenomenon has something to do with high T_c superconductors.

Several mechanisms have been advanced as possible explanations for the remarkable behavior of high T_c superconductors. In this paper, we shall explore the resonating valence bond (RVB) (2), (3) model in the slave boson formulation. We will subsequently treat a mean-field theory to explain some of their peculiar characteristics.

THE RESONATING VALENCE BOND (RVB) THEORY

Experimental data on insulating (undoped) La_2CuO_4 show that the Cu^{2+} is an $S = 1/2$ orbitally nondegenerate state with the Cu: $3d_{x^2-y^2}$ orbital strongly hybridizing with the oxygen O: $2p_x, 2p_y$ orbital in the Cu-O plane. Anderson hypothesized that the insulating state of pure La_2CuO_4 is the resonating valence bond state. According to this hypothesis, there is a fermi liquid

state with lower energy than the antiferromagnetic order wherein the nearest neighbor electrons tend to form singlet pairs. These singlet pairs resonate among various singlet configurations (and thus the name resonating valence bond) (2).

The RVB state is a liquid because it has quantum transport of spin excitations. Anderson reasoned out that, although ordinarily, the ground state configuration of systems with large quantum fluctuations is antiferromagnetic, there are pre-existing spin singlet pairs in the RVB state which become charged superconducting copper pairs by strong enough doping.

When applied to superconductors, the nearly half-filled strongly correlated Hubbard model may be appropriate. In fact, the source of high T_c superconductivity may be due to the spin correlations induced by a superexchange mechanism between electrons on the nearest neighbor lattice sites.

The starting point of the RVB model is the nearly half-filled Hubbard Hamiltonian (5)(6)(7):

$$H = -t \sum_{\langle ij \rangle \sigma} (C_{ij}^{\dagger} C_{j\sigma} + \text{h.c.}) + \mu \sum_i \eta_{i\alpha} \eta_{i\beta} - \mu \sum_{i\sigma} \eta_{i\sigma} \quad (1)$$

where $c_{i\sigma}^{\dagger}$ ($C_{i\sigma}$) is the electron creation (annihilation) operator, t is something like a transfer integral, U is the Hubbard potential and $\eta_{i\sigma}$ is the number density operator. The chemical potential μ is introduced for the doping process. Inasmuch as the site energy is measured from the chemical potential, it is normally set equal to zero. The first term in the above Hamiltonian describes a system of free band electrons while the second term characterizes the strong onsite coulomb repulsion of two opposite spin electrons.

Generally, a many-body Hamiltonian contains a one-particle kinetic energy operator and a two-body potential energy operator. A complete set of Wannier functions could be used as a basis for second quantization. A single-band Hamiltonian that could be possibly constructed is:

$$H = \sum_{ij, \sigma} C_{i\sigma}^{\dagger} \langle ij | T | j \rangle C_{j\sigma} + \frac{1}{2} \sum_{ijRl, \sigma\sigma'} C_{i\sigma}^{\dagger} C_{j\sigma'}^{\dagger} \langle ij | V | Rl \rangle \times C_{l\sigma} C_{R\sigma'} \quad (2)$$

where

$$\langle i | T | j \rangle \equiv \int d^3r w^*(\vec{r} - \vec{R}_i) T(\vec{r}) W(\vec{r} - \vec{R}_j),$$

and

$$\langle ij | V | Rl \rangle \equiv \int d^3r \int d^3r' w^*(\vec{r} - \vec{R}_i) w^*(\vec{r}' - \vec{R}_j) V(\vec{r}, \vec{r}') w(\vec{r} - \vec{R}_l) w(\vec{r}' - \vec{R}_l).$$

In the above relations $W(\vec{r} - \vec{R}_i)$ are the single-band Wannier functions. Hamiltonian introduced a constant potential $U = \langle ii | V | ii \rangle$ as the only non-vanishing component of the two-body potential. Furthermore, by using the kinetic energy function in the tight-binding approximation

$$\langle i | T | j \rangle = \epsilon \delta_{ij} - t \delta_{(ij)},$$

where ϵ is the site energy, $\delta_{(ij)} = 1$ for (ij) nearest neighbors bands and zero otherwise, the single-band Hubbard Hamiltonian takes the form

$$H = \epsilon \sum_{i\sigma} \eta_{i\sigma} - t \sum_{\langle ij \rangle > \sigma} (C_{i\sigma}^+ C_{j\sigma} + C_{j\sigma}^+ C_{i\sigma}) + \mu \sum_i \eta_{i\uparrow} \eta_{i\downarrow}, \quad (3)$$

where \sum is a sum over nearest-neighbor bands, while $\sigma = (\alpha, \beta) = (\uparrow, \downarrow)$ is a spin index. The last term tells us that the two electrons with opposite spin experience a strong repulsive force when they are in the same site. When $\epsilon = \mu = 0$, the Hamiltonian simply represents a system of free band electrons. In the event that $\mu \gg t$, each electron will localize itself at each site in order to avoid the strong repulsive force. This is what happens in a Mott insulator where each site is populated by an electron of spin 1/2. Equation (1) then is obtained by setting lattice site energy to zero while introducing the chemical potential for the doping process.

After using canonical transformations, we get an effective Hamiltonian defined in the non-doubly occupied subspace

$$\begin{aligned}
 H = & -t \sum_{\langle ij \rangle \sigma} (1 - \eta_{i-\sigma}) C_{i\sigma}^+ C_{j\sigma} (1 - \eta_{j-\sigma}) \\
 & + \mu \sum_{i\sigma} C_{i\sigma}^+ C_{i\sigma} + J \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \eta_i \eta_j), \quad (4)
 \end{aligned}$$

where we introduced the antiferromagnetic spin coupling constant $J = 4t^2/U$ and the spin angular momentum operators \vec{S} . The above Hamiltonian is too difficult to handle. For all practical purposes, we use the hopping approximation to rewrite the effective Hamiltonian as (7)

$$\begin{aligned}
 H = & -t \delta \sum_{\langle ij \rangle \sigma} (C_{i\sigma}^+ C_{j\sigma} + \text{h.c.}) + \mu \sum_{i\sigma} C_{i\sigma}^+ C_{i\sigma} \\
 & + J \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \eta_i \eta_j), \quad (5)
 \end{aligned}$$

where δ is the hopping parameter. In the insulating phase ($\delta = \mu = 0$) and with the restriction that we confine our system to a half-filled band in the singly-occupied site subspace, we get the Heisenberg antiferromagnetic Hamiltonian,

$$H_0 = J \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \eta_i \eta_j). \quad (6)$$

This describes an exactly half-filled band Mott insulator in a simple square lattice. Normally, this H_0 ground state configuration is the antiferromagnetic order. Anderson showed that by introducing the boson singlet operator,

$$b_{ij}^+ \equiv \frac{1}{\sqrt{2}} (C_{i\sigma}^+ C_{j\beta}^+ - C_{i\beta}^+ C_{j\sigma}^+), \quad (7)$$

the Hamiltonian H_0 could be rewritten as

$$H_0 = - J \sum_{\langle ij \rangle} b_{ij}^+ b_{ij} \quad (8)$$

From this, Anderson hypothesized that there is an RVB state that is a quantum liquid state in which the spins form singlet pairs rather than the long-range antiferromagnetic order.

It was shown that by starting from (8) the development of the RVB correlations and the subsequent superconducting order in the high T_c oxide superconductors could be described by a U(1) lattice gauge theory. In fact, the insulating state ($\sigma = \mu = 0$) has an almost local gauge symmetry which is spontaneously broken at low temperatures. This results in superconductivity.

With the expression of the valence bond singlet operator in (7), the effective Hamiltonian now is of the form

$$H = -t \sum_{\langle ij \rangle \sigma} (C_{i\sigma}^+ C_{j\sigma} + \text{h.c.}) - J \sum_{\langle ij \rangle} b_{ij}^+ b_{ij} + \mu \sum_{i\sigma} C_{i\sigma}^+ C_{i\sigma} \quad (9)$$

The negative sign in the second term of this Hamiltonian suggests that the singlet objects are approximate bosons which could undergo Bose condensation into zero center-of-mass momentum state. Every superconductivity practitioner knows that this state triggers superconductivity. We shall see this in detail when we discuss the RVB mean-field theory.

THE SLAVE (AUXILIARY) BOSON FORMULATION

Since the development of the resonating valence bond model of Anderson for high temperature superconductivity, several works have been made extending the said model. It was shown that in the RVB state, there exist three kinds of particle excitations: charged boson solitons which we now call holons, neutral fermion solitons which we now call spinons and true

electrons or holes. In this section we shall review (7), (9) a mathematical formalism treating these particles initially as simple mathematical objects. Later we shall render some physical explanations to show that they could be physically observable particles.

Let us consider a lattice site i in a lattice of electrons. In this site are associated four possible quantum states, namely: $|0\rangle$, $|\alpha\rangle$, $|\beta\rangle$, and $|\alpha\beta\rangle$. These states correspond with an empty site, a spin up state for an electron, a spin down state and a double occupied site (two electrons in one site) respectively. These four possible states associated with an electron lattice site form a completeness relation. Consequently, any operator associated with an electron at a particular site could be expanded as a linear combination of the above states. Thus

$$\sum_p |ip\rangle\langle ip| = |0\rangle\langle 0| + |\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta| + |\alpha\beta\rangle\langle\alpha\beta| = 1, \quad p = (0, \alpha, \beta, \alpha\beta). \quad (10)$$

This is quite interesting in the light of the mapping

$$|ip\rangle\langle ip| \longrightarrow [\text{fermion - boson operators or like combinations}]. \quad (11)$$

Thus associated with the possible states in an electron site are internal projection component operators in which a physical electron could be imagined to be constituted of. For example, for each particular state of $|ip\rangle\langle ip|$, we have such associations as

$$\begin{aligned} |0\rangle\langle 0| &\longrightarrow e_i^+ e_i & |\alpha\rangle\langle\alpha\beta| &\longrightarrow S_{i\alpha}^+ d_i \\ |\alpha\rangle\langle\beta| &\longrightarrow S_{i\alpha}^+ S_{i\beta} & |\beta\rangle\langle 0| &\longrightarrow S_{i\beta}^+ e_i, \text{ etc.} \end{aligned} \quad (12)$$

An interesting property of the projection operators associated with the four possible states in an electron lattice site is that some follow the commutator algebra while others follow the anticommutator one. For example, projection operators associated with the states $|0\rangle$ and $|\alpha\beta\rangle$ follow the commutation rules while that for $|\alpha\rangle$ and $|\beta\rangle$, the anticommutation relations. Our mapping then convinced us that e_i and d_i must be bosonic fields satisfying the commutation rules:

$$\begin{aligned}
 [e_i, e'_j] &= \delta_{ij} & [e_i, e_j] &= [e_i^+, e_j^+] = 0 \\
 [d_i, d'_j] &= \delta_{ij} & [d_i, d_j] &= [d_i^+, d_j^+] = 0,
 \end{aligned} \tag{13}$$

while the $S_{i\sigma}$ follows the anticommutation relations:

$$[S_{i\sigma}, S_{j\sigma'}^+]_+ = \delta_{ij} \delta_{\sigma\sigma'}, \quad [S_{i\sigma}, S_{j\sigma'}]_+ = [S_{i\sigma}^+, S_{j\sigma'}^+]_+ = 0. \tag{14}$$

and are therefore fermions. It is obvious that different particle operators commute.

As pointed out earlier, an operator associated with the physical electron could be constructed out of a linear combination of $|ip\rangle \langle ip|$. In particular, for the electron annihilation operator, we have

$$C_{i\sigma} \longrightarrow |0\rangle \langle 0| + \sigma |-\sigma\rangle \langle \sigma\beta|,$$

where $\sigma = \alpha, \beta$ or **(1,-1)** Transcribed in terms of boson and fermion fields this is

$$C_{i\sigma} = e_i^+ S_{i\sigma} + \sigma S_{i-\sigma}^+ d_i, \tag{15}$$

and for the electron creation operator,

$$C_{i\sigma}^+ = S_{i\sigma}^+ e_i + \sigma d_i^+ S_{i-\sigma}. \tag{16}$$

Based on equations (15) and (16) we can conjecture that a physical electron could be a composite object.

Certainly, the $C_{i\sigma}$ 'S follow the anticommutation rules:

$$[C_{i\sigma}, C_{j\sigma'}^+]_+ = \delta_{ij} \delta_{\sigma\sigma'}, \quad [C_{i\sigma}, C_{j\sigma'}]_+ = [C_{i\sigma}^+, C_{j\sigma'}^+]_+ = 0. \tag{17}$$

But for these to be satisfied, the following constraint must be imposed:

$$e_i^+ e_i + d_i^+ d_i + \sum_{\sigma} S_{i\sigma}^+ S_{i\sigma} = 1. \quad (18)$$

The mapping (11) shows that (18) corresponds with the completeness relation (10).

A straightforward calculation of the current densities associated with the $(e_i, d_i, S_{i\sigma})$ - fields by using the effective Hamiltonian shows that the total charge densities of the e_i and d_i fields could completely account for the charge of the physical electron. This implies that spinons $(S_{i\sigma})$ have neutral charge. The spin, on the other hand, could be assigned to the spinon so the charged e_i - fields (holons) and the d_i - fields are spinless.

We can thus associate the e^+ - operator to create an empty site while d^+ creates a doubly occupied site. It is also possible to associate a fundamental charge called S-charge on top of the electric charge. Thus, e^+ carries one unit of positive S-charge while d^+ carries one unit of negative S-charge $(-e_s)$. The spinon has zero S-charge (7) (9).

The important role being played by the slave boson fields could be clarified by looking at the symmetries of the effective Hamiltonian. The original Hubbard Hamiltonian (1) is invariant under a global or phase transformation of the electron field $C_{i\sigma}$. This U(1) global symmetry corresponds with the fermion number conservation. On the other hand, this Hubbard Hamiltonian has also a global SU(2) symmetry in spin space (7).

Replacing the physical electron operator with the holon and spinon operators through the slave boson transformation, and subsequently getting rid of the doublons, we will find that this effective Hamiltonian still carries the global U(1) symmetry of the original Hubbard Hamiltonian. In fact, we can make the symmetry local (by putting in a spacetime variation in the global parameter) and still find the new effective Hamiltonian invariant. Because of this symmetry we can associate a charge conservation otherwise known as the S-charge.

The local SU(2) symmetry, however, is not carried by the original Hamiltonian as well as the new effective Hamiltonian. It is the exchange parts of both Hamiltonians, H_o (the Heisenberg antiferromagnetic Hamiltonian) that is found to be local SU(2) invariant.

A MEAN-FIELD RVB THEORY IN THE SLAVE BOSON FORMULATION

We can get an effective spinon-holon Hamiltonian from the original Hubbard Hamiltonian by making use of the slave boson transformations, (7)

$$\begin{aligned}
 H = & -t \delta \sum_{\langle ij \rangle \sigma} (e_i e_j^+ S_{i\sigma}^+ S_{j\sigma} + \text{h.c.}) \\
 & - \frac{1}{2} J \sum_{\langle ij \rangle} b_{ij}^+ b_{ij} + \mu \sum_i e_i^+ e_i.
 \end{aligned} \tag{19}$$

The above Hamiltonian has been simplified by imposing the constraint relation (18) and simultaneously throwing the doublons. The first term is the coupling between the holon kinetic energy and the spinon kinetic energy with a coupling strength $t\delta$; it likewise represents the spinon-holon scattering term with a large matrix element for the localized spinon-holon scattering. Comparing this with the single-band Hubbard Hamiltonian might mislead us into concluding that the spinons will Bose condensate because the b_{ij} 's in (19) are proportional to $S_{i\sigma} S_{j-\sigma}$. But there is something in (19) that will prevent this from occurring, that is, the coupling of the holon kinetic energy with the spinon kinetic energy.

The spinon-holon Hamiltonian (19) is a direct offshoot of the slave boson transformation. It is indeed very revealing to see if the spinon-holon scattering term could prevent the Bose condensation of spinons. A mean-field RVB theory to this effect will shed light on this interesting aspect.

A mean-field theory is basically something like a classical approximation. Operationally, this is roughly done by transforming the model Hamiltonian into momentum space and subsequently making a Hartree-Fock factorization. This closely resembles the scheme of getting fermionic ensemble averages in accordance with Wick's Theorem. In our case, we simply identify the appropriate order parameters and then apply Bogoliubov transformation to diagonalize the resulting BCS-like Hamiltonian.

BZA (5) have done this for the Hubbard Hamiltonian (9). In momentum space they obtained

$$\begin{aligned}
 H = & \sum_{\vec{R}\sigma} (\epsilon_{\vec{R}} - \mu) C_{\vec{R}\sigma}^+ C_{\vec{R}\sigma} - J \sum_{\vec{R}} (\Delta \sum_{\vec{R}} C_{\vec{R}}^+ C_{-\vec{R}} + \text{h.c.}) \\
 & + N (\Delta^2 + P^2),
 \end{aligned} \tag{20}$$

where the self-consistent order parameters were defined as $\Delta = \sqrt{2} \langle b_{ij} \rangle$ and $p = \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle$. In two dimensions the ϵ_R and γ_R were found to be related as $\gamma_R = -\epsilon_R (2t\delta + pJ)^{-1}$ with $\epsilon_R = -(2t\delta + pj) (\cos k_x a + \cos k_y a)$. Observe that (k_x, k_y) are the components of the wave vector \vec{k} and a is the square lattice constant. After Bogoliubov transformation, the quasiparticle energy spectrum was obtained as well as the resulting expressions for the gap and chemical potential relations. In the insulating phase wherein $\delta = \mu = 0$, and with the order parameter p being likewise set to zero, the excitation energy spectrum becomes $E_R = \Delta \tau x |\gamma_R|$. From here on, BZA obtained the RVB state originally constructed by Anderson. In addition, BZA confirmed the existence of a pseudofermi surface and at the same time obtained a linear temperature dependence of the low temperature specific heat (because of the fermionic nature of the quasiparticles). In short, a satisfactory mean-field theory for the treatment of copper oxide-based superconductors was developed.

Let us do a mean-field theory, but this time in the slave boson formulation. We rewrite the electron operators in momentum space as

$$\begin{aligned} C_{R\sigma}^+ &= e_R S_{R\sigma}^+ + \sigma S_{R-\sigma} d_R^+ \\ C_{R\sigma} &= S_{R\sigma} e_R + \sigma d_R S_{R-\sigma}^+ \end{aligned} \quad (21)$$

In an exactly parallel way we get rid of all terms that count in the doublon operator since we do not want the doubly occupied sites. Consequently, we have

$$\sum_{R\sigma} C_{R\sigma}^+ C_{R\sigma} = \sum_{R\sigma} e_R e_R^\dagger \sum_{\sigma'} S_{R\sigma}^+ S_{R\sigma'} ; C_{R\sigma}^+ C_{R+\beta}^+ = e_R e_{R+\beta} S_{R\sigma}^+ S_{R+\beta}^+$$

On the other hand, the order parameters could be conveniently chosen as

$$\Delta_R = \left\langle e_R^+ e_{R'} \sum_{\sigma} S_{R\sigma} S_{R'-\sigma} \right\rangle ; P_R = \left\langle e_R e_{R'} S_{R\sigma}^+ S_{R'\sigma} \right\rangle ,$$

so that we could finally obtain the effective Hamiltonian in momentum space as

$$\begin{aligned}
 H = \sum_R (\epsilon'_R - \mu) e_R e_R^+ - \frac{1}{2} J \sum_R \Delta_R \gamma'_R e_R e_{-R} \\
 + N (\Delta_R^2 + P_R^2). \quad (22)
 \end{aligned}$$

In the first term of the above Hamiltonian we used the equation of constraint $\sum_{\sigma} S_{R\sigma}^+ S_{R\sigma} = 1 - e_R^+ e_R$. The expressions for the site energy ϵ'_R and γ'_R are proportional to those obtained in the conventional way. The second term clearly tells us that the holons will bose condensate into zero momentum state as they form an effective cooper pair. In effect, this holon condensation will trigger the superconductivity of the system.

We mentioned previously that the apparently deceptive form of the effective Hamiltonian (19) might yield the unexpected bose condensation of the spinons but that the scattering term would prevent this from occurring. Actually, this has been convincingly shown in a recent work by Baskaran where he formulated the problem by using functional integrals.

Our treatment is a bit different. We observed first all that the choices of our order parameters couples the holon kinetic energy and the spinon kinetic energy. This is true for both order parameters. In effect, we trivialized the so-called scattering term and, hence, ignored the dynamics involved in it. This is one reason why the Hamiltonian given by (22) is expressed entirely in terms of the holon field. The constraint equation (18) also contributed to this.

CONCLUSION

A mean-field theory for the RVB in the slave boson formulation was also tried by some authors. The theoretical framework runs like this. Starting from the original Hubbard Hamiltonian a

$$\begin{aligned}
 H = -t \sum_{\langle ij \rangle \sigma} (C_{i\sigma}^+ C_{j\sigma} + \text{h.c.}) + \mu \sum_i \eta_{i\alpha} \eta_{i\beta} - \mu \sum_{i\sigma} \eta_{i\sigma}, \quad (23)
 \end{aligned}$$

slave boson formulation was used together with the constraint $\eta_e + \eta_d + \eta_s = 1$. By eliminating the e^+ , d^+ , ed terms perturbatively as in the Rice transformation and, furthermore, by projecting out the pure d -terms ($\eta_d = 0$) he got the projected Hubbard model

$$H = t \sum_{\langle ij \rangle \sigma} e_i^+ e_j S_{j\sigma} S_{i\sigma}^+ + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad (24)$$

where $\vec{S}_i \vec{S}_j$ are spin operators written purely in terms of the spinon operators. Attempting a mean-field theory with a mean value of $\langle e_i^+ e_j \rangle$ and $\langle S_{i\sigma}^+ S_{j\sigma} \rangle$, he obtained an effective spinon Hamiltonian,

$$H = t \sum_{\langle ij \rangle \sigma} \langle e_i^+ e_j \rangle S_{j\sigma}^+ S_{i\sigma} + J \sum_{\sigma\sigma'} \langle S_{i\sigma}^+ S_{j\sigma'} \rangle S_{i\sigma}^+ S_{j\sigma'}. \quad (25)$$

This gives unphysical results because it leads to spinon propagators of the order of $1/t$ and $1/J$.

Our comment runs like this: Maybe it should have been better if the original Hubbard Hamiltonian is used and not worked with the canonically transformed Hubbard Hamiltonian, and then used the slave boson formulation to obtain

$$H = -t \delta \sum_{\langle ij \rangle \sigma} (e_i^+ e_j S_{i\sigma}^+ S_{j\sigma} + \text{h.c.}) - \frac{1}{2} J \sum_{\langle ij \rangle} b_{ij}^+ b_{ij} + \mu \sum_i e_i^+ e_i,$$

with $b_{ij} = \sum_{\sigma} S_{i\sigma}^+ S_{j\sigma}$. An appropriate choice of the order parameters might lead to the phenomenon of holon condensation and obtain expressions for propagators that are physically acceptable.

Of course, there are also some drawbacks in our formulation. For one thing, we almost entirely ignored the dynamics of the spinon operators and instead concentrated on the holon field. Our motivation for this is understandable. We wanted an effective Hamiltonian that will describe holon condensation and this is what we exactly did.

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