SOCIAL SCIENCES

THE EFFECTS OF LANGUAGE FORMATION MATHEMATICAL LEARNING AND PERFORMANCE: AN INFORMATION-PROCESSING PERSPECTIVE

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ABSTRACT

Using an information-processing perspective, the paper addresses the question on the effect of using a bilingual's first or second language on learning and doing math. The perspective assumes that math performance can be analyzed into smaller components, and that language factors can affect these components in different ways. The paper integrates the results of 16 experiments, each of which studies the effects of language format on six components of word problem solving using different experimental paradigms in cognitive psychology. Four possible effects of language format on the components were considered and the results of the experiments were compared to these hypothesized effects. The varying effects of language on word problem solving performance were integrated in a proposed "language-routing model" which specifies how language factors effects specific components of math performance.

Keywords: *Abilities, Bilingualism; Cognition; Education; Information Processing; Language; Learning; Mathematics; Problem-Solving; Thinking.*

INTRODUCTION

What is the effect of using a bilingual's first or second language on bilingual's mathematical learning and performance? The question about the relationship between language and mathematics has both theoretical and practical significance. At the theoretical level, the question relates to a larger issue regarding the relationship between language and thought, an issue that has long concerned psychologists, anthropologists, and philosophers, among others. At the time practical level, the question relates to issue regarding the language of instruction especially in bilingual and multilingual communities. In the Philippines, research on the relationship between language and mathematics has been undertaken by researchers in psychology, linguistics, and education. These researches typically involve the comparisons of instructional materials and/or procedures in Filipino or English using unitary measured of mathematical achievement or ability.

In my own research I have been using the information-processing approach to study the effect using Filipino or English on mathematical learning and performance. In this paper, I summarize the findings of a series of 16 cognitive psychological experiments that look into the effects of using either English or Filipino on the operation of specific components of mathematical skills among Filipino-English bilinguals.

RELATING LANGUAGE AND MATHEMATICS

One of the most compelling hypotheses about the relationship between language and thoughts was articulated by Whorf (1956). He proposed the linguistic determinism hypothesis which states that the structures of language determine the structures of thought. That is, how we perceive, experience, and think about the world depends on the elements of the language we use to talk about the world. This rather strong prescription has long been discredited, particularly with the increase in scientific knowledge about language and about thought. Recent empirical evidence seems to favor two other positions: (a) the structures of thought determine the structures of language (Gerrig & Banaji, 1994) and (b) structural variations in language result to parallel variations in thought; actually, a weaker version of linguistic determinism (Hunt & Banaji, 1991).

The experiments I will describe has been derived from this weaker version of linguistic determinism. In particular, the conceptual framework assumes that using either Filipino or English might result to variations in mathematical performance among Filipino-English bilinguals. The goal in the series of experiments was to determine the specific effects of using either language on particular aspects of mathematical skill.

At a very general level, we can propose four possible effects of using Filipino and English on mathematical performance. First, there could be a *null effect* that is, there is no relationship between the language used and mathematical performance. It is conceivable that mathematical activity is such an abstract process that it is not affected by contextual factors like language. Second, there could also be a *language-proficiency effect;* that is, mathematical performance might depend on the person's proficiency in the language being used. Therefore, mathematical performance is better when the task is rendered in the person's more fluent language. Third, there could be a *language-of-learning effect;* that is, performance could be better when the task is rendered in the language that was used in acquiring the concepts and procedures. It is possible that math performance is better when the tasks are rendered in the language one has become used to. Finally, there could be a *structural-fit effect;* that is, the structures of one language are more adequate in handling the processing of mathematical concepts and operations. Some people who argue that mathematics should and could only be taught in English are actually implicitly assuming that the English language is structurally more adequate than Filipino for processing mathematical information.

THE INFORMATION-PROCESSING APPROACH: DEFINING THE COMPONENTS OF MATHEMATICAL SKILL

There are many different ways psychologists try to understand and explain cognitive functioning, including mathematical performance. However, cognitive psychologists use an information-processing approach to study the different cognitive functions. This approach assumes that any cognitive activity involves the transformation of information through a series of processes (procedures or operations) that also draw upon existing knowledge representations (or schemata). The analogy is often made between cognitive functions and computer programs. Both include a series of procedures (routines, subroutines, etc.) that operate on information that is encoded or "inputted") and that also draw from internal memory representations.

An important element of this assumption is the notion that any cognitive function could be described as consisting of a complex of smaller functions, which could be further described as having even smaller functions. These smaller elements of a cognitive function are called components and subcomponents. These components and subcomponents are procedures and schemata that operate on specific input. The output of specific components and subcomponents could feed into other components, and so on. The operation of all the component and subcomponent parts are related in specific ways that make one cognitive activity.

Using the information-processing approach, we can, therefore, look at mathematical skill as being composed of smaller components and subcomponents. So rather than thinking of mathematical skill as a monolithic, generalized skill, we can think of it as being a composite of several specific but interrelated skills.

If we conceive of mathematical skill in this way, we can then think of language as having a different specific effect on each component, depending on the character of the component. In my research program, I have been working on the assumption that the effect of using Filipino or English in mathematical activities is not a unitary effect. It is conceivable and very possible that the use of either language would have different specific effects depending on the particular component of mathematical skill.

The experiments I describe look into six components of mathematical skill. These are (a) problem-text comprehension, (b) problem-model construction, (c) retrieval of specialized meanings, (d) retrieval of number facts, (e) retrieval of relevant analogous problem information, and (f) application of analogous problem information. These six components could be seen as parts of two very related task: solving math word problems and transferring information between analogous math word problems. The relationships among the different components are specified in detail theories of word problem solving (see e.g., Kintsch & Greeno, 1985), and analogical transfer (see e.g., Ross, 1984, 1989).

The Effects of Language on Components of Mathematical Skill

To study the effects of language on the six different components of mathematical skill, different cognitive tasks were used to target the particular components. The task was then presented in either Filipino or English to Filipino-English bilingual subjects. The specific effects were then assessed by comparing performance in the Filipino and the English tasks. In this paper, the description of the specific methods used and the results found in the different experiments will be rather brief. I refer the interested readers to the specific papers for more detailed accounts of the methodology and of the results.

Problem-Text Comprehension

When given math word problems to solve, a person's first task is to read the text and develop an understanding of the information being described in the text. This is the component of problem text comprehension which results to the construction of the text base. In a way, this is a straightforward reading task. The text base is later used as the basis for constructing the abstract problem model.

I studied problem text comprehension in two experiments by using a recall paradigm (Bernardo, 1996a, 1997). Beginning arithmetic students were given simple word problems in either Filipino or English and later asked to recall the problems. The assumption is that the manner of recall will be based on the students' comprehension of the different story elements. I found that recall was reliably better in the students' first language, whether Filipino or English. Error analysis further indicated that students had more gross comprehension errors in their second language. It seems that students comprehend the problem text better in their first language. Moreover, consistent with the view that problem solving begins with this reading process, students were also better in solving the problems when it was in language in which they comprehended it better. Therefore these results show a language-proficiency effect: Students were better at comprehending problems in their first and more fluent language.

Problem-Model Construction

From the text base, the word problem solver constructs a representation of the structural elements of the problems (i.e., the quantitative relations, etc.). This representation is called the problem model and is used as the basis for formulating the solution.

To study the effects of language on problem-model construction, I used a problem completion paradigm in two experiments (Bernardo, 1996a, 1997). Proficient problem solvers were given Filipino and English arithmetic word problems without the final question. They were asked to provide a question for each incomplete problem. The assumption was that if the problem solver correctly constructed the problem model, he or she would be able to supply the appropriate question based on the information specified in the problem model and using knowledge about prototypical problem models. I found that there was no difference in the subjects' ability to complete the problems in Filipino and in English. It seems that subjects were able to construct the problem model just as effectively in either language, regardless of what their first language was, and despite the fact that the subjects were taught word problem solving in English and using English word problems.

Therefore, these results show a null effect. It seems that as far as problem-model construction was concerned, language factors have no effect. These results suggest that language does not seem to affect the more abstract elements of math word problem solving.

However, subjects with English as first language performed this task better than those whose first language was Filipino. I think this reflects an interactive effect of language proficiency and language of learning. That is, it may be that those who are more proficient in English learn the process of problem-model construction more effectively because the students are given word problems in English. Even if the process that needs to be learned is essentially abstract, the students can only encounter this information by first routing it through language. Hence, the student who has better facility with the route will learn the abstract processes better. Once they have learned the abstract material, then the language route does not matter anymore, as shown be the earlier results.

Retrieval of Specialized Meanings

One specific subcomponent of constructing problem models in the task of interpreting words and phrases in specialized ways. For example, the phrase, "gave three candies to" can be translated as "plus three" in the problem model. A word like "more" also takes a very specific meaning in problem model construction in math: "more by a specified amount".

In three experiments (Bernardo, 1996b), I studied the extent to which this task was entrenched by comparing interpretations of words like "more" and "less" in mathematical context (i.e., in algebra problems) and a non-mathematical context (i.e., in a story). The results of two experiments showed that the retrieval of specialized meanings was stronger in the language in which subjects learned word problem solving (i.e., English). The results of one experiment showed no difference between English and Filipino, however, this experiment used a procedure quite different from the other two.

These results suggest a language-of-learning effect. It seems the task of retrieving specialized meanings of words is more entrenched in the language in which this task was learned and more often done. Since the subjects encountered math word problems only in English, the specific skill seems to have been better established for English problems. However, the inconsistency in the results warns us to accept this conclusion with some caution.

Retrieval of Number Facts

In computing for the solutions to the word problems, the proficient problem solver has to retrieve basic number facts like "1 + = 4" from memory. Young or novice problem solvers arrive at the answer using strategies other than memory retrieval. However, research (e.g., LeFevre, Bisanz, & Mrkonjic, 1988; Zbrodoff & Logan, 1990) has shown that among people with high numeracy skills, the answers to simple addition and substraction operations are derived using highly automated memory retrieval processes.

In five experiments (see e.g., Bernardo, 1996c), I studied the effect of language format on the retrieval of number facts using a verification paradigm. Subjects were presented a pair of addends followed by a number. The formats of this information (numerical, verbal Filipino, or verbal English) were varied across blocks of trials for the addends and within blocks for the last number. Subjects were asked to verify whether the number that followed the addends was the sum of the addends. Their verification times were measured. The results show that processing number facts in numerical and verbal-English format was highly automated (fast and flat verification times and low error rates), but processing in verbal-Filipino format was very inefficient (slow and variable verification times and high and variable error rates).

The results show a language-of-learning effect. It seems that the subjects have automated processing number facts in English because that is the language that they have most often processed the information. Again, it is possible that the effect is a structural-fit effect; the English format items actually require shorter articulation time. So it may be that processing in English is faster simply because of the brevity of the items. However, the difference in articulation of the English and Filipino number words does not seem to be commensurate to the difference in verification times nor is it consistent with the error rates. It is possible however, that the brevity of English number words makes it the preferred format in practice, hence the effect is still consistent with a language-of-learning effect.

Retrieval of Analogous Problem Information

One effective strategy for solving a problem is to use a solution that worked in an earlier analogous problem. Since the problems have analogous problem models, the solution that worked in one should work for the other. The first component of this analogical problem solving strategy is the retrieval of the relevant analogous problem.

In two experiments (Bernardo, 1994a, 1996d) I studied the retrieval of analogous word problems in basic probability using a reminding paradigm. Subjects were given a set of word problem to study either in Filipino or English. These problems included explanations about the relevant principles, equations, and solutions. They were then given a set of analogous test problems in Filipino and English, which they were asked to solve after being given some cue about the relevant analogous problem. The subject's solutions were scored in terms of whether they showed the retrieval of the appropriate analogous problem information. The results showed that subjects retrieved the analogous problem better if the language formats of the old and new problems were the same. That is, if they were given a test problem in Filipino, they were more likely to retrieve the relevant problem if this was studied also in Filipino. There was no overall advantage of studying or testing the problems in either language.

The results of the experiments show a language-of-learning effect. In this case, the subjects were more likely to retrieve relevant information when the language used for learning matched the language used for testing.

Application of Analogous Problem Information

Analogical problem solving does not only involve the retrieval of the relevant analogous information. The problem solver must also be able to correctly apply or instantiate the information from the old problem to the new problem information.

In two experiments (Bernardo, 1996d) I studied whether language affected the application of analogous information by using a modified reminding paradigm. The procedure described in the previous section was used except that the required equations were provided. The task that remained was to apply the information to the new problems. The results showed that subjects performed equally well for the Filipino and English study and test problems. Once they have the relevant information, they can apply the information to any language format. These results show a null effect of language. It seems that the application of abstract concepts and procedures is not affected by language factors.

TRENDS AND CONCLUSIONS

The results of sixteen experiments on six components of mathematical word problem solving reveal different effects of language factors. First, I found null effects on two components that involve abstract concepts and procedures: problem-model construction and the use of analogous problem information. Both these procedures operate at that level where the problem solver already has access to the abstract structural elements of the problem. It seems, therefore, that as regards the use of abstract problem elements, language factors have no effect.

Second, I found a language-proficiency effect in the component of problem text comprehension. The structural information in math word problems is couched in linguistic format, and the problem solver first needs to operate linguistic processes before the structural elements can be activated. As far as this process is concerned, the problem solver's effectiveness will depend on his or her proficiency with the language of the problem. These findings are most consisting with earlier research demonstrating the effects of semantic and syntactic factors in problem comprehension (e.g., Cummins, Kintsch, Reusser, & Weimer, 1988; De Corte & Verschaffel, 1987; De Corte, Verschaffel, & De Win, 1985).

Third, I found language-of-learning effects in three components: retrieval of specialized meanings, retrieval of number facts, and retrieval of analogous problem information. Notice that all these components involve the retrieval of information from existing knowledge representations. It seems that the retrieval of this information is very much dependent on some match between the language during retrieval and the language used in the process of developing these representations. This finding is most consisting with research demonstrating how learners use contextual information in developing abstract problem representations (Bernardo, 1994b) and how the activation of such information is hyperspecific to the problem context (Bernardo & Okagaki, 1994).

Finally, I found an interactive effect of language proficiency and language-of-learning in problem-model construction. It seems that students whose first language matched the language in learning or teaching the component of problem-model construction acquired the component skill better that those for whom these two factors did not match (i.e., those whose first language was not English). Again, consistent with earlier research (Bernardo, 1994), the route to learning abstract information involves language processing. The learner's success in learning to some extent depends on his or her facility with language processes.

The most glaring finding in these series of studies is that there is no unitary effect of language factors on mathematical performance. This should serve as a strong cautionary statement for those who have been or who plan to make sweeping statements about the role of language in mathematical performance, mathematical learning, or mathematical teaching. The specific effect language factors depends on the character of the particular components of mathematical performance.

However, I do not wish to give the impression that the varied effects of language on math performance cannot be understood in a coherent frame. For the purpose of providing a tentative organizing theoretical framework, I propose a routing model with the following general and specific propositions:

(1) Mathematical concepts and procedures are essentially abstract in character, however, most of these concepts and procedures can only be learned by couching them in linguistic structures.

(2) Components related to learning are affected by language factors, but those which operate independent of the learning components are not.

 (2.1) Early learning of these concepts and procedures is dependent on the learner's proficiency in the language used to convey these concepts and procedures.

(2.2) Because of encoding-specificity in retrieval, activation of learned concepts and procedures is more efficient in the language context in which these concepts and procedures were learned.

(2.3) The operation or use of concepts and procedures which are separate from retrieval processes are not affected by language factors.

This routing model is an attempt to develop a cognitive psychological framework for understanding the relationship between language and mathematical skill. I am still currently conducting more experimental studies to validate, modify, and fine tune this model. Hopefully, this model will not only be of theoretical interest to those of us who study the human mind, but it should also be of use to those who have a stake in the development of the human mind.

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